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Local nearrings of order at most 31

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Nearrings are a generalization of rings in the sense that the addition in nearrings need not be commutative and only one distributive law holds. A nearring R is called local, if the set L of all non-invertible elements of R forms a subgroup of its additive group R^+.

The nearring library of the package "SONATA" [1] of the computer algebra system GAP 4.6.4 [2] contains all nearrings up to order 15 and all nearrings with identity up to order 31.

Recall that an abelian group is a group of type $(p^k, p^l, \dots, p^m)$ if it is isomorphic to a direct sum of cyclic p-groups of orders p^k, p^l,..., p^m, respectively, with prime p and integers $k, l, \dots, m$. Let C_n denote a cyclic group of order n.

Proposition 1 *Let R be a local nearring of order at most 31, which is not a nearfield. Let $n(G)$ be the number of all non-isomorphic local nearrings whose multiplicative group is isomrphic to G. The following statements are hold: 1) G is isomorphic to group C_4 and $n(G) = 4$. 2) G is isomorphic to group $(2,2)$ and $n(G) = 7$. 3) G is isomorphic to group C_6 and $n(G) = 1$. 4) G is isomorphic to group $(4,2)$ and $n(G) = 227$. 5) G is isomorphic to group $(2,2,2)$ and $n(G) = 114$. 6) G is isomorphic to group $(6,2)$ and $n(G) = 4$. 7) G is isomorphic to group $(6,3)$ and $n(G) = 14$. 8) G is isomorphic to group C_{20} and $n(G) = 1$. 9) G is isomorphic to symmetric group S_3 and $n(G) = 2$. 10) G is isomorphic to group $C_3 \times S_3$ and $n(G) = 15$. 11) G is isomorphic to quaternion group Q_8 and $n(G) = 48$. 12) G is isomorphic to dihedral group D_8 and $n(G) = 236$. 13) G is isomorphic to alternative group A_4 and $n(G) = 9$. 14) G is isomorphic to Miller-Moreno group $(C_3 \times C_3) \rtimes C_2$ and $n(G) = 4$. 15) G is isomorphic to Miller-Moreno group $C_5 \rtimes C_4$ and $n(G) = 1$. 16) G is isomorphic to group $C_5 \rtimes C_4$ with nontrivial center and $n(G) = 3$.*

1. Aichinger E., Binder F., Ecker Ju., Mayr P. and Noebauer C. SONATA — System of Nearrings and their Applications, Version 2.6, Johannes Kepler Universitaet Linz, 2012 (http://www.algebra.uni-linz.ac.at/Sonata/).

2. The GAP Group, Aechen, St Andrews. GAP — Groups, Algorithms and Programming, Version 4.6.4, 2013 (http://www.gap.dcs.st-and.ac.uk/ gap).