Jürgen Prestin

Periodic Multiresolution

Institute of Mathematics, Lübeck University, Lübeck, Germany
E-mail: prestin@math.uni-luebeck.de

It is the aim of this lecture to present some ideas of multiscale analysis and time-frequency-localization for spaces of periodic functions.

In particular, we consider finite dimensional nested spaces of trigonometric polynomials constructed from de la Vallée Poussin means of the Dirichlet kernel. Following an approach of C.K. Chui and H.N. Mhaskar we investigate the corresponding Multiresolution Analysis. The scaling functions and wavelets are given explicitly as trigonometric fundamental interpolants and decomposition and reconstruction algorithms can be described in simple matrix notation. The circulant structure of all relevant matrices allows the use of Fast-Fourier-Transform techniques for the actual implementation. Thus we achieve almost optimal complexity compared to other wavelet approaches derived from implicit two-scale relations, while dealing with a fully computable trigonometric multiresolution analysis with explicit algebraic formulas.

Furthermore we describe the multivaraite setting and discuss different approaches including tensor products and Boolean sums.

The special structure of the underlying de la Vallée Poussin means allows to transform most of these results to the algebraic case. In particular we obtain algebraic polynomial wavelet bases for $C[-1,1]$, where certain interpolation conditions for the wavelets are satisfied.